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RADIATION PROPAGATION IN MULTI-LAYER SYSTEMS WITH VARIABLE OPTICAL PROPERTIES

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Abstract—Relationships are obtained of propagation and attenuation of radiation fluxes in real multi-layer systems of selectively absorbing and scattering materials with variable optical properties. Boundary reflection, monisotropic scatter and changes in the space distribution of radiation intensity are included into the consideration.

NOMENCLATURE

k, σ, absorption and scattering coefficients of a material layer element [m^{-1}];

Λ, $= \dfrac{\sigma}{k+\sigma}$, probability of photon survival (scattering criterion);

$\bar{k}, s, \varepsilon_{ef}$, averaged coefficients of absorption, backward scatter and extinction of the layer element [m^{-1}];

Λ_{ef}, $= \dfrac{s}{\bar{k}+s}$, mean effective probability of photon survival (the Schuster number);

τ, optical depth;

$\rho_{i,k}$, boundary reflection coefficient;

γ, constant factor;

β, ν, material or imaginary numbers;

E, radiation flux density [$\mathrm{W/m^2}$];

E_0, $= E^+ + E^-$, space irradiance [$\mathrm{W/m^2}$];

q, $= E^+ - E^-$, net flux density [$\mathrm{W/m^2}$];

w, $\bar{k}E_0$, radiation energy absorbed per unit time by volume element [$\mathrm{W/m^3}$];

R, T, reflectivity and transmittance of a layer with finite thickness l;

R_∞, reflectivity of a layer with infinite optical thickness;

λ, wavelength.

Subscripts

λ, spectral;

in, incident;

ef, effective;

i, n, k, layer numbers.

Academician A. V. Luikov when solving in his fundamental works [11]–[15] the heat-transfer problem in engineering processes was the first who evaluated the significance of i.r.-radiation and optical properties of irradiated materials [11]. In what follows the main results are presented of the works inspired by Luikov.

In engineering processes various means of irradiation are used: two-side and one-side irradiations, that with diffuse and directional or mixed radiation fluxes.

During i.r.-radiation optical properties of materials undergo changes [6–8] since their physical and chemical properties do not remain the same. With i.r.-radiation by integral flux the spectral radiation composition within the material changes due to multiple scatter effects on optical nonuniformities that may result in changed integral absorption and scattering coefficients (see Figs. 2 and 3).

Studies of radiative heat transfer [1] and energy transfer in multilayer systems [8, 9, 18, 19, 21] become very important because real objects under i.r.-radiation are systems consisted of several layers of selectively absorbing and scattering materials (multi-layer shields and coatings of meteorological and space instruments and apparatuses, units of buildings, outside coatings of vegetal materials, biological objects, food stuffs, etc.).

In view of the above, knowledge of the relationships of propagation and attenuation of integral radiation fluxes in real materials (single-layers and multi-layers) with constant and variable optical properties with account for boundary reflection, selective absorption and scatter is very important.

Development of efficient calculation methods of heat and mass transfer processes [11–15] involving i.r.-radiation based on multi-layer systems with moving boundaries of phase conversions is also of importance for practical usage in different fields of science and technology.

Works [18, 21, 24] are concerned with particular problems for a pile of nonscattering layers [24], a weakly scattering thin film between metallic layers [21], a system of scattering layers and two-layer purely scattering spherical atmosphere [18] with boundary reflection neglected. Works [5–7, 10, 16–18, 22, 23] deal with energy transfer of monochromatic radiation in the atmosphere and different scattering media with absorption and radiation coefficients variable along the coordinate [17, 18]. Some attempts have been made to account for changes in the scattering indicatrix [10, 17, 23], scattering particle concentrations [7], optical anisotropy in inhomogeneous media, dependence of absorption [5–7] and scattering coefficients [6] on the incident flux density (nonlinear optics).

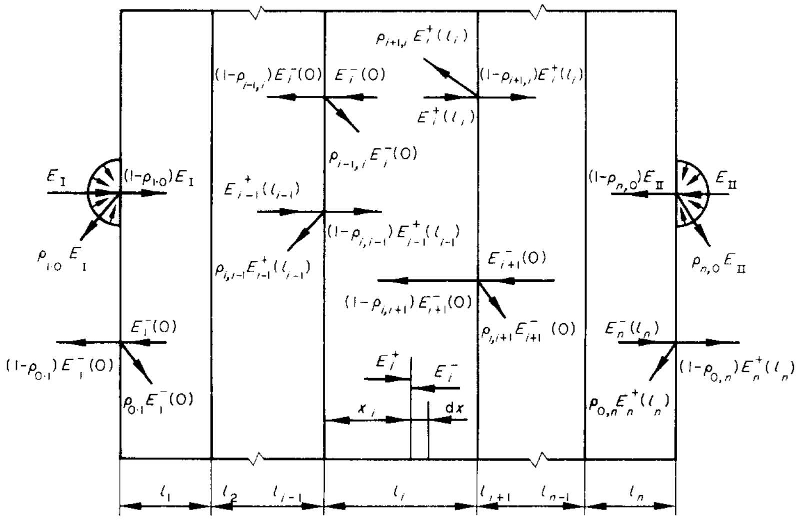

FIG. 1. Radiant fluxes in multi-layer systems of selectively absorbing and scattering materials.

Optical properties of materials under solar and i.r.-radiation in thermoradiation installations are independent of the incident flux intensity, but rather depend on physico-chemical properties and their changes during irradiation. In this case both coefficients, $\bar{k}(x)$ and $s(x)$, characterizing optical properties of materials, which can absorb and scatter radiation, change across the layer.

The methods suggested in [9] including boundary reflection into the boundary conditions with the boundaries expressed as fictitious asymmetrically reflecting and transmitting layers give rather simple relationships of propagation and attenuation of radiation flux in multi-layer systems by the differential-difference technique.

In works [1–4] and [6–9] constants of absorption $\bar{k} = mk$ and backward scattering $s = m\delta_s\sigma$ averaged over the half-space are assumed independent of coordinate x.

Moreover, for most of real objects nonisothermal scatter on large particles occurs when indicatrix is far advanced [8, 16], therefore changes of the space distribution of radiation intensity distribution $B(\omega', x)$ over the depth x should be accounted for in the solution even in case of diffuse irradiation of a layer system.

In accordance with the more accurate differential-difference method presented later, accounting for changes in intensity $B(\omega', x)$ with coordinate x, the optical properties of the ith layer (Fig. 1) are characterized by parameters of absorption $k(x_i)$ and backward scattering $s(x_i)$ averaged over the half-space. These are related with optical characteristics k and σ independent of x by the equalities

$$\bar{k}(x_i) = m(x_i)k_i, \quad s(x_i) = m(x_i)\delta_s(x_i)\sigma_i. \tag{1}$$

Parameters m and δ_x variable over x_i which describe the indicatrix shape and variations of the irradiation conditions of elementary layer dx with the coordinate may be calculated from the available formulas [1, 7, 8].

In this case for any ith layer with thickness l_i $(0 \leqslant x_i \leqslant l_i)$ of a multi-layer system (Fig. 1), a system of linear differential equations with two variable coefficients $\bar{k}(x_i)$ and $s(x_i)$ for the densities of opposite semi-spherical fluxes E_i^+ and E_i^- is obtained similar to that with constants $\bar{k}$ and s [1–4], [6–8].

By simple transformations with x_i substituted by the optical depth

$$\tau_i = \int_0^{x_i} [\bar{k}(x_i) + s(x_i)]\,\mathrm{d}x, \tag{2}$$

layer thickness l by optical thickness $\tau_{l,i}$ determined from (2) at $x_i = l_i$ and with the Schuster number Λ_{ef} [1, 8] which is an effective probability of photon survival in an elementary scattering process

$$\Lambda_{\mathrm{ef}}(x_i) = \frac{s(x_i)}{\bar{k}(x_i) + s(x_i)} = \frac{\delta_s(x_i)\Lambda}{1 - [1 - \delta_s(x_i)]\Lambda}, \tag{3}$$

we obtain for the ith layer the new equation system with one variable coefficient $\Lambda_{\mathrm{ef}}(\tau_i)$ for E_i^+ and E_i^-

$$\left.\begin{aligned} \frac{\mathrm{d}E_i^+}{\mathrm{d}\tau_i} &= -E_i^+ + \Lambda_{\mathrm{ef}}(\tau_i)E_i^- \\ \frac{\mathrm{d}E_i^-}{\mathrm{d}\tau_i} &= -E_i^- + \Lambda_{\mathrm{ef}}(\tau_i)E_i^+ \end{aligned}\right\} \quad 0 \leqslant \tau_i \leqslant \tau_{l,i}. \tag{4}$$

The boundary conditions for system (4) may be written according to Fig. 1 and [9] in the form of (2), (3) [9] or (22), (23) [9]. Solution of system (4) for a general case of variable $\Lambda_{\mathrm{ef}}(\tau_i)$ will be found later in equations (19), (21), (22).

It should be pointed out that simpler solutions of (4) are obtainable. For a number of real materials of the third and fourth groups classified in accordance with their optical properties [8] in case of solar and high-temperature i.r.-radiation, Λ_{ef} in (4) may be assumed constant, independent of τ_i, as it is slightly changing with δ_s when $\sigma > k$. For the ith layer of a system of such materials (shielding coatings, wood, cocoon coatings, bread, grain, fruit candy, starch, powdered sugar, vermiculite, gypsum, fired clay, chamotte, abrasives, foam plastics, powders and coatings of MgO, Al_2O_3, $MgCo_3$, etc.) $\Lambda_{ef} > 0{\cdot}9$ [8] in a short-range spectrum, and a 1·5-fold change of δ_s results in the deviation of Λ_{ef} from the original value within 0·5 per cent. It is also known from experimental data [16] that the backward scattering indicatrix is close to the diffuse one even for a very far advanced indicatrix. Thus, practically in all the cases of scatter on small and large particles the expected deviation from δ_s is within 50 per cent as far as the radiation changes from directional toward diffuse. In case of diffuse irradiation the change in δ_s will obviously be smaller.

On the basis of the above experimental data $\Lambda_{ef}(\tau_i)$ in (4) may be substituted by constant effective photon survival probability $\Lambda_{ef,i}$, being mean over the ith layer. In this case changes in the irradiation conditions of layer dx and the scattering indicatrix are taken into account with the aid of variable τ_i dependent on $m(x_i)$, $\delta_s(x_i)$, k_i and σ_i.

General solution of system (4) for the ith layer including boundary reflection and changes in the space distribution of the radiation intensity with boundary conditions (2), (3) [9] is of the form

$$\begin{aligned} E_i^+(x_i) = {} & \frac{E_{i,I}C_{i,i-1}}{1-B_{i-1,i}B_{i+1,i}\psi_i^2} \\ & \times[\exp(-K_i\tau_i)-B_{i+1,i}\psi_i^2\exp(K_i\tau_i)] \\ & +\frac{E_{i,I}C_{i,i+1}\psi_i}{1-B_{i-1,i}B_{i+1,i}\psi_i^2} \\ & \times[\exp(K_i\tau_i)-B_{i-1,i}\exp(-K_i\tau_i)] \end{aligned} \tag{5}$$

$$\begin{aligned} E_i^-(x_i) = {} & \frac{E_{i,I}C_{i,i-1}\psi_i}{1-B_{i-1,i}B_{i+1,i}\psi_i^2} \\ & \times\{\exp[K_i(\tau_{l,i}-\tau_i)]-B_{i+1,i}\exp[-K_i(\tau_l-\tau_i)]\} \\ & +\frac{E_{i,II}C_{i,i+1}}{1-B_{i-1,i}B_{i+1,i}\psi^2} \\ & \times\{\exp[-K_i(\tau_{l,i}-\tau_i)]-B_{i-1}\psi_i^2 \\ & \times\exp[K_i(\tau_{l,i}-\tau_i)]\} \end{aligned} \tag{6}$$

with the following notations

$$K_i = (1-\Lambda_{ef,i}^2)^{\frac{1}{2}}, \quad \psi_i = R_{i\infty}\exp(-K_i\tau_{l,i}). \tag{7}$$

Coefficients $C_{i,i\pm1}$ and $B_{i\pm1,i}$ describing boundary reflection of the ith layer are defined by the equalities [9]

$$C_{i,i\pm1} = \frac{1-\rho_{i,i\pm1}}{1-\rho_{i\pm1,i}R_{i\infty}}, \quad B_{i\pm1,i} = \frac{R_{i\infty}-\rho_{i\pm1,i}}{R_{i\infty}(1-\rho_{i\pm1,i}R_{i\infty})}. \tag{8}$$

Heat flux densities $E_{i,I}$ and $E_{i,II}$ are related with E_I and E_{II} which irradiate the whole multi-layer system by expressions (9)–(12) [9].

Equations (5), (6) give radiation fluxes $q(x_i)$, as well as $E_0(x_i)$ and $\omega(x_i)$ at a depth of x_i in a multi-layer system with account for reflection effects at the boundaries of the ith layer and changes in space intensity distribution $B(\omega' x)$. They allow general formulas for thermoradiation characteristics of the layer adjacent to different layers (media) within a multi-layer system to be obtained.

Transmittance and reflectivity of the ith layer may be found from expressions (5) and (6) with $E_{i,II} = 0$ as

$$T_i = \frac{(1-\rho_{i+1,i})}{1-\rho_{i+1,i}R_{i\infty}}\,\frac{C_{i,i-1}(1-R_{i\infty}^2)\exp(-K_i\tau_{l,i})}{1-B_{i-1,i}B_{i+1,i}\psi_i^2} \tag{9}$$

$$R_i = \rho_{i,i-1}+(1-\rho_{i-1,i})C_{i,i-1}R_{i\infty} \times\frac{1-B_{i+1,i}\exp(-2K_i\tau_{l,i})}{1-B_{i-1,i}B_{i+1,i}\psi_i^2}. \tag{10}$$

With constant δ_s and m we find from (1) that $\tau_l = (\bar{k}+s)l$, then $K\tau_l = Ll$ and formulas (11), (12) agree completely with those obtained earlier in [9]. In a particular case of a single layer in the air (vacuum) with $\rho_{i,i-1} = \rho_{1,0}$, $\rho_{i,i+1} = \rho_{0,1}$ and constant δ_s and m formulas (9), (10) completely agree with the available expressions [3, 8, 9].

In a general case of integral flux radiation optical properties of real multi-layer systems depend on coordinate x and may be characterized by absorption, $k(x)$ and scattering, $\sigma(x)$, coefficients related with $\bar{k}$ and s by equalities

$$\bar{k}(x_i) = m(x_i)K(x_i); \quad s(x_i) = m(x_i)\delta_s(x_i)\sigma(x_i). \tag{11}$$

Simple manipulations on (4) give a new system of linear differential equations with one variable coefficient $\Lambda_{ef}(\tau_i)$ for net flux density $q_i = E_i^+ - E_i^-$ and space irradiance $E_{0i} = E_i^+ + E_i^-$

$$\frac{dq_i}{d\tau_i} = -[1-\Lambda_{ef}(\tau_i)]E_{0i}, \quad \frac{dE_{0i}}{d\tau_i} = [1+\Lambda_{ef}(\tau_i)]q_i. \tag{12}$$

The boundary conditions for the above system may be written within the suggested method of layer summation with account for boundary reflection by representation of the boundaries as fictitious non-absorbing layers [9] having asymmetrical reflectance $\rho_{i,k} \neq \rho_{k,i}$ and transmittance $t_{i,k} = 1-\rho_{i,k} \neq t_{i,k} = 1-\rho_{k,i}$. The ith layer under consideration is expressed as a combination of three layers: its two boundaries, namely, the $(i, i-1)$th layer; the $(i, i+1)$th layer and the ith layer itself with reflectance R_i and transmittance T_i determined from formulas resultant from the solution of (12), boundary reflection being neglected [8].

It may be assumed that onto the ith layer considered flux densities $E_{i,1} = E_i^+(0)$ and $E_{i,2} = E_i^-(\tau_{l,i})$ fall from two sides. These densities are related with the external flux densities $E_{i,I} = E_{i-1}^+(\tau_{l,i-1})$ and $E_{i,II} = E_{i+1}^-(\tau_{i+1} = 0)$ at the boundaries of the layer by

$$E_{i,1}(\tau_i = 0) = (1-\rho_{i,i-1})E_{iI}+\rho_{i-1,i}E_i^-(0) \tag{13}$$

$$E_{i,2}(\tau_i = \tau_{l,i}) = (1-\rho_{i,i+1})E_{i,II}+\rho_{i+1,i}E_i^+(\tau_{l,i}). \tag{14}$$

The boundary conditions for (12) may be written with (13), (14) as

$$\text{at } \tau_i = 0, \quad \begin{aligned} E_0(\tau_i = 0) &= (1+R_i)E_{i,1}+T_iE_{i,2} \\ q(\tau_i = 0) &= (1-R_i)E_{i,1}-T_iE_{i,2} \end{aligned} \tag{15}$$

$$\text{at } \tau_i = \tau_{l,i} \quad \begin{aligned} E_0(\tau_{l,i}) &= T_iE_{i,1}+(1+R_i)E_{i,2} \\ q(\tau_{l,i}) &= T_iE_{i,1}-(1-R_i)E_{i,2}. \end{aligned} \tag{16}$$

Transformation of system (12) for q_i or E_{0i} with arbitrary function $\Lambda_{ef}(\tau_i)$ gives the self-conjugated differential equation of the second order

$$\frac{d}{d\tau_i}\left[\frac{1}{1-\Lambda_{ef}(\tau_i)}\frac{dq_i}{d\tau_i}\right]-[1+\Lambda_{ef}(\tau_i)]q_i=0, \qquad (17)$$

whose explicit solution may only be obtained with function $\Lambda_{ef}(\tau_i)$ known.

For the materials listed earlier in the paper in a general case of irradiation of the ith plane layer with integral fluxes from two sides, the function $\Lambda_{ef}(\tau_i)$ may be expressed as

$$\Lambda_{ef}(\tau_i)=1-\frac{a_i}{\tau_i+b_i}-\frac{a_i}{C_i-\tau_i} \qquad (18)$$

where a, b, c are constants dependent on the optical properties of the material and spectral composition of incident flux densities $E_{i,1}$ and $E_{i,2}$ of integral radiation.

With (18) the substitution of new variable $\xi(\tau_i, \Lambda_{ef,i})$ for τ_i in (17) results in the equation of the Bessel type [13] of which solution is represented by the cylindrical function

$$q=Z_\nu(\beta\xi^\gamma)=Z_{2ai}\left[2\left(\frac{2a}{b+c}\right)^{\frac{1}{2}} i\sqrt{\{(\tau+b)(c-\tau)\}}\right] \qquad (19)$$

expressed in terms of the modified νth order Bessel functions of the first ($I_\nu(Z)$) and second ($K_\nu(Z)$) kinds.

For the considered practical cases of irradiation and materials with sufficiently strong scatter ($R_\infty > 0{\cdot}5$; $\Lambda_{ef} > 0{\cdot}8$) the first terms of the Bessel function series may only be taken (19) (at large z) [13]. The general solution in case of irradiation from two sides becomes rather simple

$$q(\tau_i)=\left\{C_{1,i}\exp\left[-2\sqrt{\left(\frac{2a_i}{b_i+c_i}\xi_i\right)}\right]+C_{2,i}\exp\left[2\sqrt{\left(\frac{2a_i}{b_i+c_i}\xi_i\right)}\right]\right\}\xi_i^{-1/4} \qquad (20)$$

where

$$\xi_i=(\tau_i+b_i)(c_i-\tau_i). \qquad (21)$$

Integration constants $C_{1,i}$ and $C_{2,i}$ in equation (20) are determined from boundary conditions (13)–(16).

From general solution (20) integral radiation fluxes in materials with variable particular cases, such as one-side irradiation ($E_2 = 0$) of a finite layer thickness τ_l and at $\tau_l \to \infty$.

In a layer with an infinite optical thickness ($\tau_l \to \infty$) the net flux density and space irradiance are determined from

$$q(\tau)=E_{in}(1-R_\infty)\left(1+\frac{\tau}{b}\right)^{-\frac{1}{4}}\times\exp\left\{-2(2ab)^{\frac{1}{2}}\left[\left(1+\frac{\tau}{b}\right)^{\frac{1}{2}}-1\right]\right\} \qquad (22)$$

$$E_0(\tau)=E_{in}(1+R_\infty)\frac{1+4\left[2ab\left(1+\frac{\tau}{b}\right)\right]^{\frac{1}{2}}}{4a\left(1+\frac{\tau}{b}\right)^{-4}}\times\exp\left\{-2(2ab)^{\frac{1}{2}}\left[\left(1+\frac{\tau}{b}\right)^{\frac{1}{2}}-1\right]\right\} \qquad (23)$$

where

$$R_\infty=\frac{4[(2ab)^{\frac{1}{2}}-a]+1}{4[(2ab)^{\frac{1}{2}}+a]+1}. \qquad (24)$$

Expressions (22) and (23) allow rather simple (without integration of analytical expressions of $q_\lambda(x)$ and $E_{0\lambda}(x)$ over the radiator spectrum) calculation of integral radiation in selectively absorbing and scattering materials with account for changes of mean integral optical properties with the coordinate in terms of function $\Lambda_{ef}(\tau)$ and optical depth τ. To this end constants $a = \bar{k}^*(0)$ and $b = \bar{k}^*(0)+s^*(0)$ of a unit thickness layer are determined by averaging $\bar{k}_\lambda$ and s_λ over the spectral composition of quantities $E_0(0)$ and $q(0)$ determined from condition (15) at $x = 0$ with the R and T of the irradiated material layer given.

Figure 2 shows that the spectral composition of space irradiance $E^*_{0x}(\lambda)$ with fixed depth x varies due to multiple scattering and selective absorption effects (compare curves 2 and 1′).

Changes of mean integral characteristics over the depth x in case of irradiation of starch by an integral flux from a lamp KG–1000 are presented in Fig. 3(a). The value of Λ_{ef} increases with depth and at large $x > 2$ mm approaches unity (pure scatter).

From the available experimental data for starch irradiated by lamps LG–1000 at $x = 0$ $\bar{k}^*(0) = 0{\cdot}641$ and $s^*(0) = 3{\cdot}862$ [8] integral $R_\infty = 0{\cdot}612$ is found from (24). This differs from integral reflectivity $R_\infty = 0{\cdot}615$ of [8] obtained by integration of $R_{\infty\lambda}$ over the whole spectrum only by 0·003 (error of $\sim 0{\cdot}5$ per cent).

For estimation of the accuracy of the obtained analytical expressions for radiation flux distribution in materials with variable coefficients $\bar{k}(x)$ and $s(x)$ (see

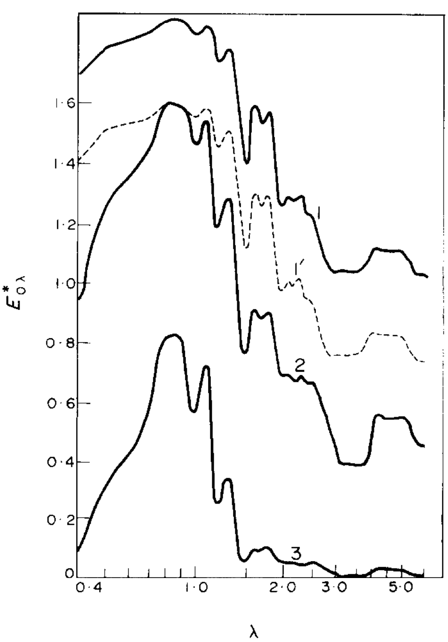

FIG. 2. Plot of space irradiance E^*_0 vs wavelength $\lambda(\mu m)$ in a semi-infinite layer of potato starch at different depths x (mm): 1, $x = 0$; 2, 0·2; 3, 1·0. 1′, curve parallel to 1.

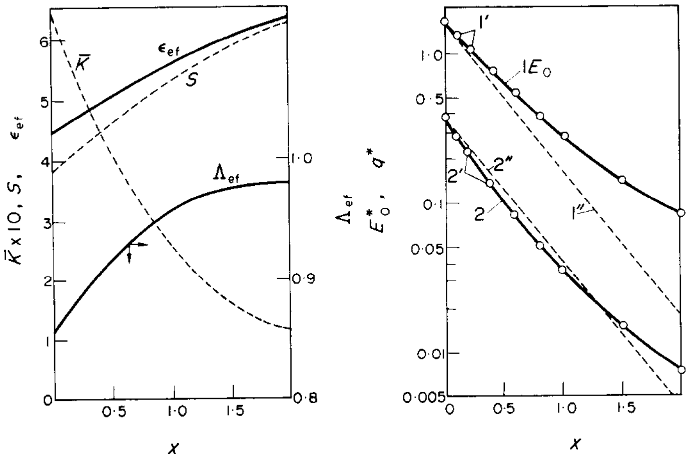

FIG. 3. Changes of integral optical characteristics (a) and radiation field E_0^* (1, 1′, 1″), q (2, 2′, 2″″) (b) calculated by different methods: 1, 2 from formulas (22), (23); 1′, 2′, numerical integration [8]; 1″, 2″, the averaged characteristics method [8].

Fig. 3(a)), $q^*(x)$ and E_0^* are evaluated from (22) and (23) in case of irradiating starch with lamp KG–1000.

Figure 3(b) demonstrates that at the layer boundary $x = 0$ and with different x, E_0^* and q^* evaluated by integration over the spectrum and from analytical relations (22) and (23) are different as average within 1·0 per cent that implies a fairly high accuracy of the suggested method for radiation energy transfer in absorbing and scattering materials with variable optical properties.

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PROPAGATION DU RAYONNEMENT DANS LES SYSTEMES A PLUSIEURS COUCHES AVEC PROPRIETES OPTIQUES VARIABLES

Résumé—Des relations sont obtenues qui donnent la propagation et l'atténuation des flux de rayonnement dans les systèmes réels à plusieurs couches de matériaux à absorption et diffusion sélectives avec propriétés optiques variables. La réflexion sur les parois, la diffusion anisotrope et les changements dans la distribution spatiale de l'intensité du rayonnement sont considérés.

DIE STRAHLUNGSAUSBREITUNG IN MEHRSCHICHTIGEN SYSTEMEN MIT UNTERSCHIEDLICHTEN OPTISCHEN EIGENSCHAFTEN

Zusammenfassung—Es werden Beziehungen hergeleitet für die Ausbreitung und Schwächung von Strahlung in mehrschichtigen Systemen aus teilweise absorbierenden und streuenden Materialien mit unterschiedlichen optischen Eigenschaften.

In den Betrachtungen wurden Grenzreflektionen, nicht isotrope Streuung und Änderungen in der räumlichen Verteilung der Strahlungsintensität berücksichtigt.

РАСПРОСТРАНЕНИЕ ИЗЛУЧЕНИЯ В МНОГОСЛОЙНЫХ СИСТЕМАХ С ПЕРЕМЕННЫМИ ОПТИЧЕСКИМИ СВОЙСТВАМИ

Аннотация — Получены закономерности распространения и ослабления потоков излучения в реальных многослойных системах селективно поглощающих и рассеивающих материалов, обладающих переменными оптическими свойствами, с учетом граничного отражения, неизотропного рассеяния и изменения пространственного распределения интенсивности излучения.